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IMPROVED TRANSMISSION OF IONS WITH INHOMOGENEOUS MAGNETIC FIELDS

S B. KARMOHAPATRO

INSTITUTE OF NUCLEAR PHYSICS, CALCUTTA.

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ABSTRACT. Design parameters for symmetrical and asymmetrical first order focussing conical shaped inhomogeneous magnetic analysers with field varying as r^{-n} where $0.5 < n < 1$ are considered for utilising their high resolving power with improvements in transmission of ions Some representative cases for such a 180°-magnetic analyser are shown with a discussion for using such magnets with different sector angles in mass spectroscopy, which will be more advantageous than the conventional flat type homogeneous magnetic analysers, when solid angle and resolving power of the instruments are simultaneously considered

Application of the inhomogeneous magnetic analysers, having field shape

$$H = H_0 \left(\frac{r_0}{r} \right)^n \qquad \ldots \quad (1)$$

derived by Kerst and Serber (1941), have been frequently used for betatron and synchrotrons and have now extended to the field of β-ray and mass spectrometry. Siegbahn and Svartholm (1946) have utilised this type of inhomogeneous magnets for two directional focussing of the β-particles at an angle $\sqrt{2}\pi$ with $n = 0.5$. Higher order focussing for improved resolution has been considered by Shull and Dennison (1947), Verster (1950), Stoker *et al.* (1954), Lee-Whiting and Taylor (1957), Judd and Bludman (1957) and others with two directional focussing magnetic analysers. Such first and second order focussing magnetic analysers are now used in β-ray spectrometry Svartholm (1951) proposed conical magnets for an average two directional focussing of the charged particles and a β-ray spectrometer by Arbman and Svartholm (1955) has been constructed on this suggestion

Two directional focussing of the charged particles with an inhomogeneous magnet having an angle $\sqrt{2}\pi$ has been considered by Judd (1950), Rosenblum (1950) and others. The possibility of using such an analyser with a shaped pole boundary was discussed by the author (Karmohapatro, 1955) in connection with the design of a mass spectrometer. Sternheimer (1952) has discussed the focussing of charged particles with such inhomogeneous magnets with proper consideration to the focussing effect caused by the fringing field of a sector magnet. In case of a magnetic analyser, in which the source and the detector are within the magnetic field, the focussing or defocussing effect due to fringing field does not arise. But for a sector magnet, even with a homogeneous flat field, axial focussing is possible in the inhomogeneous fringing field region as shown by Camac (1951) and Cross (1951).

Sternheimer (1952) derived a generalised expression determining the parameters of a first order focussing inhomogeneous magnetic analysers with proper consideration to the fringe-field focussing. Though the aim of this derivation was mainly for reducing the distance of source and detector from the pole boundary of a two directional fringe-field focussing magnetic analyser by introducing a slight inhomogeneity in the magnet with a field index $n < 0.5$, the scope of the work, being a generalised treatment, is extensive and is applicable to such inhomogeneous magnetic analysers with any field index $n \leqslant 1$.

If we consider the case of a symmetrical sector shaped inhomogeneous magnetic analyser having field index n, sector angle ϕ, and ϵ being the angle made by the central beam with the normal at the entrance and exit of the pole boundary, the radial focussing of the charged particles is possible at a distance

$$l = \frac{(1-n)^{\frac{1}{2}} \cot (1-n)^{\frac{1}{2}}\phi + \tan \epsilon + (1-n)^{\frac{1}{2}} \operatorname{cosec} (1-n)^{\frac{1}{2}}\phi}{(1-n) - \tan^2\epsilon - 2(1-n)^{\frac{1}{2}} \cot (1-n)^{\frac{1}{2}}\phi \tan \epsilon} \tag{2}$$

where l is also the distance of the source in unit of the equilibrium orbit r_0, which is hereafter used in all cases indicating any length. This formula is directly derived from Sternheimer's expression (1952) This formula reduces to

$$l = \frac{\cot \phi + \tan \epsilon + \operatorname{cosec} \phi}{1 - \tan^2 \epsilon - 2 \cot \phi \tan \epsilon} \quad \ldots \quad (3)$$

when $\quad n = 0$

and $\quad l = \cot \phi + \operatorname{cosec} \phi \quad \ldots \quad (4)$

when $\quad n = 0, \quad \epsilon = 0.$

The above formulae are due to Herzog (1949) and applicable to homogeneous flat type magnetic analysers extensively used in mass spectroscopy.

For $\epsilon = 0, \quad n \neq 0$ exp. (3) reduces to

$$l = \frac{1}{(1-n)^{\frac{1}{2}}} \left[\cot (1-n)^{\frac{1}{2}}\phi + \operatorname{cosec} (1-n)^{\frac{1}{2}}\phi\right] \quad \ldots \quad (5)$$

This is the case derived for an asymmetric inhomogeneous magnet with field index n as treated by Judd (1950) and Rosenblum (1950) in connection with the two directional focussing of charged particles with $n = 0.5$

The axial focussing of the charged particles also occurs at a distance

$$l' = \frac{n^{\frac{1}{2}} \cot n^{\frac{1}{2}}\phi - \tan \epsilon + n^{\frac{1}{2}} \operatorname{cosec} n^{\frac{1}{2}} \phi}{n - \tan^2 \epsilon + 2n^{\frac{1}{2}} \cot n^{\frac{1}{2}} \phi \tan \epsilon} \quad \ldots \quad (6)$$

for a symmetrical magnetic analyser. This expression reduces to

$$l' = \frac{2-\phi \tan \epsilon}{2 \tan \epsilon - \phi \tan^2 \epsilon} \qquad \ldots \quad (7)$$

for $n = 0$ and it is equivalent to the expressions derived by Camac (1951) and Cross (1951) for focussing of charged particles with the fringing field of a homogeneous sector magnet.

It should be mentioned here that a positive real value of l' cannot be found out for the axial focussing alone with an inhomogeneous symmetrical magnetic analyser.

It is interesting to note that the dispersion of the charged particles with inhomogeneous magnetic analysers of field index n is $\frac{1}{1-n}$ times greater than the homogeneous ones This advantageous property has been utilised for achieving double dispersion along with higher transmission with a two directional focussing magnetic analyser having $n = 0.5$ as described by Siegbahn and Svartholm (1946).

It is evident that for $n = 0.8$ or $n = 0.9$, the dispersion will respectively be 5 or 10 times greater than the homogeneous magnetic fields The Russian authors Alseevesky and Prudkovsky (1955) have constructed mass spectrometers with field index $n = 0.8$ and 0.9. Dubroniv and Balabina (1955) have constructed one which has a resolving power of $\sim 10^5$ and can measure atomic masses to an accuracy of $\sim 10^{-6}$. Since velocity focussing electrostatic analysers are absent and electrometer detection can be used instead of photographic plates, the instrument becomes simpler

It should be mentioned that since $n > 0.5$, axial focussing is not attained. The resolving power R of a mass spectrometer is expressed as

$$R = \left(\frac{M}{\Delta M} \right)_{max} = \frac{D}{A}$$

where M is the average mass of the two masses and ΔM is their difference, D is the dispersion of the magnetic analyser and A is the width of the beam consisting of aberation due to the analysers, velocity dispersion of ions, etc

The solid angle of such an inhomogeneous magnetic analyser is

$$\frac{\Omega}{A} = \frac{1}{\left[l^4 + \frac{1+l^2}{n(1-n)} \right]^{\frac{1}{2}}} \text{ sterad} \qquad \ldots \quad (9)$$

where Ar_0^2 is maximum available cross-section area for the ion path.

We now consider expression (5) for radial focussing of ions with a magnetic analyser having $n > 0.5$. The graphs (figure 1) determining the values of l with

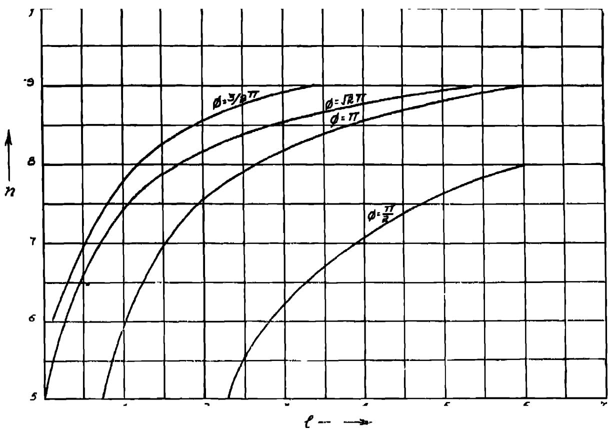

Fig. 1. Relations between the object distance and field index n for an inhomogeneous symmetrical magnetic analyser with different sector angles

different sector angle ϕ and $n = 0.8$ or 0.9 show that l increases with higher values of n. For the same n-value l decreases for higher values of ϕ.

The resolving power of such a magnetic analyser can on principle be increased without reducing intensity by reducing the aberration A, which consists of the terms as follows :

$$A = a + b + r_0\alpha^2 + \text{velocity dispersion etc.} \tag{10}$$

where a and b are entrance and exit slit widths and $r_0\alpha^2$ is the second order aberration due to the magnetic analyser, α being the half divergence angle. The term $r_0\alpha^2$ can be reduced to the third order by shaping the pole boundaries of sector homogeneous magnet as suggested by Kerwin (1949) and Hintenberger (1949). For a given resolving power, reduced aberration gives an advantage of increasing a and b for attaining a better transmission. For inhomogeneous symmetrical magnetic analysers having $n = 0.8$ or 0.9 such second order focussing with a straight pole boundary is also possible. The condition for such a focussing with the ions, not incident normally to sector boundary, is expressed as

$$l = -\frac{1}{3\tan\epsilon} = \frac{2\cot(1-n)^{\frac{1}{2}}\phi/2}{3(1-n)^{\frac{1}{2}}} \tag{11}$$

$$\tan\epsilon = -\tfrac{1}{2}(1-n)^{\frac{1}{2}}\tan(1-n)^{\frac{1}{2}}\phi/2 \tag{12}$$

With this expression different parameters of an analyser for $n = 0.8$ and 0.9 have been calculated and given in Table 1.

The table shows also a reduction in the value of l compared to figure 1 in which perpendicular incidence of ions is considered. Thus expressions (9) and (10) show that this type of magnet will give us a two-fold advantage for attaining higher transmission

This is to a certain extent off set by the low dispersion attained with these type of magnets in comparison to those with perpendicular incidence of ions.

Moreover, this method will not be very helpful in practice, since the axial defocussing effect due to the fringing field, may destroy the improved focussing effect. This difficulty is also encountered in case of the homogenous magnets However, Kerwin (1950) has constructed such a homogeneous sector magnet and attained proper second order focussing of ions. In case of the inhomogeneous magnets, there is also the possibility of utilizing the parameters of Table 1 to construct a suitable magnet for second order focussing of ions with high dispersion.

TABLE I

n	ϕ	ϵ	l
0 9	$3\pi/_2$	8°17′	2 275
0 9	$\sqrt{2}\pi$	−7°38′	3 623
0 9	π	−4°53′	3 92
0 9	$\pi/_2$	2°17′	8.426
0 8	$3\pi/_2$	−21°25′	82
0 8	$\sqrt{2}\pi$	10°43′	1 753
0 8	π	−8°54′	1 163
0 8	$\pi/_2$	−4°18′	4 026

Another way to improve transmission of ions with these high dispersion magnets lies in reducing the value of l without axial defocussing by utilising the focussing effect due to fringing field of the inhomogeneous sector magnet. Unfortunately combining expressions (2) and (6), suitable parameters for a symmetrical magnetic analyser with radial and axial focussing cannot easily be found out. It is worthwhile to consider the cases of asymmetric sector magnetic analysers having conical shape and the same first order focussing effect as described by Sternheimer

(1952). The schematic diagram of such an analyser is shown in figure 2 and it satisfies the following conditions for a first order radial focussing.

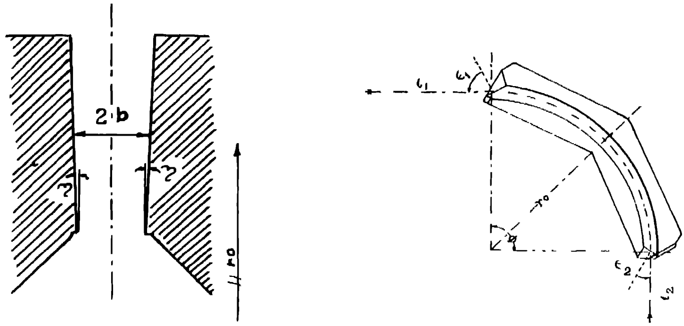

Fig. 2 (*a*) End view of the pole face of the conical magnet showing the parameters which determine the field index n.

(*b*) Side view of the magnetic analyser showing the parameters for focussing of ions

$$2\ \tan \epsilon_2 = \frac{(1-n)^{\frac{1}{2}}}{\cot\,[(1-n)^{\frac{1}{2}}\,\phi + x]} - \frac{n^{\frac{1}{2}}}{\cot[n^{\frac{1}{2}}\phi + y]} \tag{13}$$

where

$$\cot x = \frac{(1-n)^{\frac{1}{2}} l_1}{1 + l_1 \tan \epsilon_1}$$

$$\cot y = \frac{n^{\frac{1}{2}} l_1}{1 - l_1 \tan \epsilon_1}$$

and

$$l_2 = \frac{\cot\,[n^{\frac{1}{2}}\phi + x]}{n^{\frac{1}{2}} + \tan \epsilon_2 \cot\,[n^{\frac{1}{2}}\phi + x]} \tag{14}$$

Here l_1 and l_2 are distance of the source and detector from the entrance and exit pole boundaries respectively, ϵ_1 and ϵ_2 are the angles made by the central beam of ions with the normals to the entrance and exit pole boundaries respectively, and ϕ, n are same as before.

A few parameters for such high dispersion asymmetric magnetic analysers for values of $n = 0.8$ and 0.9 are calculated and some representative cases are shown in figures 3 and 4,.

Under the same conditions implied in the Sternheimer's findings, any sector angle can be considered with $0.5 < n < 1$ for high dispersion as well as better transmission by suitable choice of the parameters

If we compare figure 1 with figures 3 and 4, it is seen that due to reduced l_1 with shaped pole boundaries, transmission increases to a great extent. As,

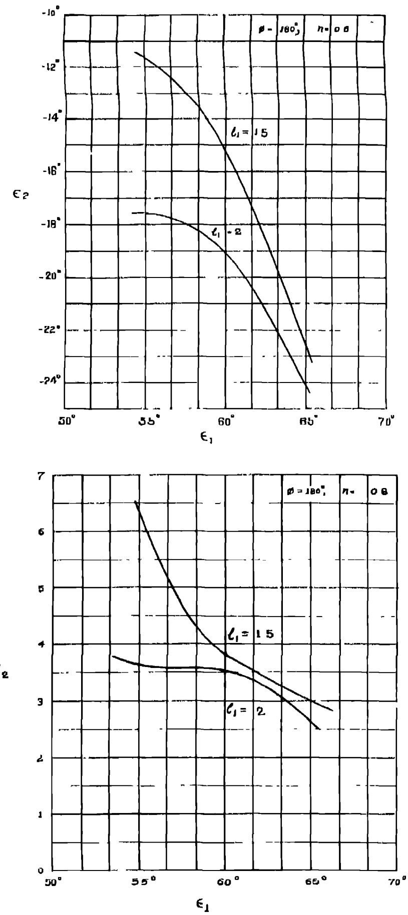

Fig 3. Relations among entrance and exit angles (ϵ_1, ϵ_2) and image distance l_2, object distance l_1, for focussing of ions with a magnet having $\phi = 180°$, $n=0.8$.

for example, with $\epsilon_1 = 60°$, a 180° annular magnetic analyser with $n = 0.9$, one will have a solid angle = 0.1183 sterad instead of 0.0249 for $\epsilon_1 = 0$, for the same available cross-section area for the ion trajectory and for 10 times better dispersion than a homogeneous magnet for both the cases.

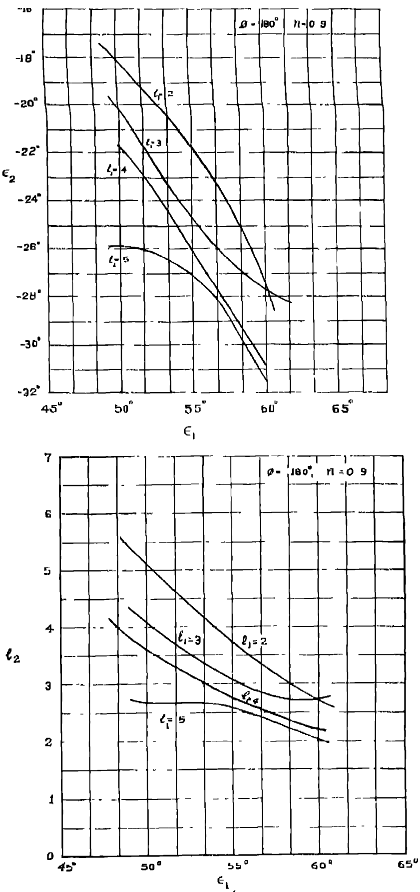

Fig. 4. Relations among entrance and exit angles (ϵ_1, ϵ_2) and image distance l_2, object distance l_1 for focussing of ions with a magnet having $\phi = 180°$, $n = 0.9$.

It is also possible to utilise the conditions of second order focussing principle due to Kerwin (1949) and Hintenberger (1949) with these types of magnet with which the resolution or the solid angle can be increased further by choice of suitable parameters and the reduced distance of the source or the detector adds some advantage to it.

For mass spectroscopes, in which high intensity ion sources are not usually used except for isotope separation, improvement in transmission of ions as suggested above will be of some importance. With suitable parameters, a moderately high resolution mass spectrometer with less complicated ion source and detection system can be designed on this principle, which will be superior to a conventional flat type instrument, when the resolving power and the solid angle are simultaneously considered

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